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Optimal Design of Generalized Batch-Storage Network Considering Shortage Costs

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Product shortages, which cause backlogging and/or lost sales costs commonly occur in the chemical industries, especially in the commodity polymer business. Shortage costs in a supply chain optimization model are studied under the framework of a generalized batch-storage network. A classical economic order quantity model with backlogging costs suggested an optimal time delay and final product delivery lot size. A product shortage can be mitigated by advancing production/transportation or by purchasing a substitute product from a third party as well as by a product delivery delay in the supply chain network. Optimal solutions that consider all means for recovering shortage are more complicated than the classical case. Four solutions are identified depending on the parametric range and variable bounds. The optimal capacity for production/transportation processes associated with a product in shortage can differ from that of a product not in shortage. An illustrative example is presented in support of the analytical results. © 2013 American Institute of Chemical Engineers *AIChE J*, 59: 2454–2470, 2013

Keywords: optimal lot-size, batch-storage network, backlogging cost, lost sales cost

Introduction

This study treats the shortage costs in a supply chain optimization model within the framework of a generalized batch-storage network. Shortages are a concern in the chemical industries. Since production lines can produce multiple products, shortages of some products and overstocking of other products are unavoidable because of inventory imbalances over the multiple products. For example, most commodity polymer plants that produce products such as high-density polyethylene (HDPE), polypropylene (PP), low-density polyethylene (LDPE), or linear low-density polyethylene (LLDPE) consist of production lines involving reaction, extrusion, and packaging stages. The reaction stage operation has the characteristics of both a batch process (one process produces multiple products in sequence), and a continuous process (the process runs continuously during a product transition as well as having a variable lot size for each product). The operating modes of the extrusion and packaging stages are semicontinuous. The scheduler must reduce the number of product transitions to reduce losses due to the sequence-dependent product changeover costs in the reaction stage. However, reducing the number of product changeovers in the reaction stage can cause overstocking of some products and shortages of other products at the final packaging stage. The consequences are an increase in the inventory holding costs of overstocked products and backlogging or lost sales costs of products in shortage. Product shortages can be prevented by insuring sufficient capacity in product storage units and by enabling multiple processes; however, increasing the capacity of storage units and the number of processes is costly. Allowing shortages of less profitable products will be more cost-effective if shortage costs can be appropriately managed.

Many chemical engineering studies of supply chain optimization include shortage costs in their models.[1–3] However, the concept of a shortage cost in chemical engineering studies does not consider each customer's situation, and models of the consumer-oriented business environment are difficult to construct. Shortage costs strongly depend on an individual customer's situation, as does the optimal solution of a supply chain network. Two groups of models are available to treat shortage costs in the operations research literature.[4] One group of these models assumes that in the event of a shortage, a customer will wait for the delivery of the next order for his demand to be fulfilled (the so-called backlogging case). In another case, the customer is not prepared to wait if there is a shortage (the so-called lost sales case). The backlogging costs are associated with the loss of a customer's good will, whereas lost sales costs are associated with demand reduction. A recent review of inventory models that include partial backlogging, in which unsatisfied demand is partially backlogged and the remaining demand is lost, referenced around 150 published articles.[5] It covers models that include, in addition to partial backlogging, additional features such as: deteriorating inventory, nonconstant demand

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patterns, an uncertain replenishment quantity, pricing and multiple items. A review of lost sales inventory theories referenced around 160 published articles.[6] Lost sales models are classified into a continuous review and a periodic review with additional topics such as: supply interruptions, emergency replenishments, multiple demand classes, order splitting, perishable items, pricing, joint replenishments and multiechelon. Note that the model introduced in this article falls into the category of multisupplier, multiproduct, multistage, nonserial, multicustomer supply chain network with reverse material/currency flows. Thus, the principal contribution to the literature of the model presented in this study is to address the general case of shortage cost in multistage production systems. We can find only three articles that address the case: two that address two stage systems[7,8] and one that considers the serial multistage case.[9] The basic assumptions involved in extending the single-stage economic order quantity (EOQ) style model to the general multistage case requires that the process be stationary (periodic), nested (coordinated), and follow a cycle time that is an integer multiple cycle time (power-of-two). Under these assumptions, an approximate, near optimal solution can be developed that is within 2% of optimality.[10] The nested (coordinated) assumption means that the phase shifts in the cycle times of multistage processes are ignored. Note that the cycle time coordination requires additional installation of automation equipment, and, thus, will be practically impossible for quite a number of processes. Cycle time length at each stage is restricted to an integer power of two multipliers of the basic period[10] or to be an integer multiple of the cycle time of the adjacent downstream stage.[11] This power-of-two (integer multiple cycle time) assumption is based on the observation that the inventory holding cost of up-stream material is lower than that of down-stream material since normally the feedstock material to a stage is cheaper than the product material. This is not true for byproducts and the price of material is not the only factor in determining the inventory holding cost. In this study, the nested (coordinated) and power-of-two (integer multiple cycle time) assumptions are relaxed.

Classical economic order quantity models with backlogging costs suggest an optimal time delay and lot size for the final product delivery.[4] Product shortages can be mitigated by advancing the production/transportation of a product, by purchasing substitute products from third parties, or by delaying delivery in the supply chain network. These operational decisions incur costs and impose limits on the variables. The consideration of all operational means for recovering from product shortage in a supply chain network suggests the existence of other possible solutions in addition to the classical EOQ model with backlogging costs. In this work, we use the assumptions of the classical EOQ model with backlogging costs and develop optimization models that include shortage costs within the framework of a generalized batch-storage network. This study was motivated by the observation that the concept of shortage costs developed in the operations research, combined with our batch-storage network design methodology, offers the possibility of more general solutions than those reported in the operations research literature and opens new research opportunities for treating consumer behavior within supply chain management.

In this study, the complex supply chain network is represented by a batch-storage network[12–21] that covers most supply chain structural components, for example, raw material purchasing, production, transportation, and finished product demand. The batch-storage network model introduced in this article extends our previous work[21] to include multitasking semicontinuous production processes which allow us to represent commodity polymer supply chain network. We refer to this extension as the generalized batch-storage network (GBSN).

This article deals with a single-period supply chain design problem. In that sense, the GBSN structure is really an approximation to the general supply chain system. The basic elements of the GBSN are production/transportation processes and material/currency storages. The processes can treat multiple materials/currencies. Therefore, GBSN can represent the features of design/planning model in detail, but is not enough to represent all of the features of a general scheduling application for supply chain. For example, the GBSN is not appropriate for production sequence optimization. Most supply chain optimization models are MILP formulations[22,23]; however, the design method developed in this study is the combination of analytic lot sizing (process capacity) solutions and a nonconvex MINLP model to compute average flow rates. The MINLP model includes frequency dependent costs compared to the MILP models. We will demonstrate how to solve the MINLP model with a linearization technique of nonlinear functions. Note that the benefit of this study exists in the analytic lot sizing solutions not in the MINLP model although the combination of both solutions generates a global optimal solution in theory. The optimality of analytic lot sizing solutions is still valid provided that the average flow rates are given by any other (linear) optimization models. The resulting simple analytic solutions can greatly enhance the proper and quick investment decision for the preliminary supply chain design problem confronted in diverse economic situations. Also note that the method in this study is restricted to a deterministic approach, and, thus, the safety stocks to consider demand forecasting errors are additionally required for real applications. The stochastic approach for the optimal design of batch-storage network to consider uncertainties in demand forecasting and batch operation is already developed.[19] It is not a nontrivial work to use the stochastic approach for GBSN with shortage costs; however, we will not consider it here for simplicity.

The remainder of this article is organized as follows. First, variables and parameters are defined and then the concept of backlogging costs is introduced. An optimization model that includes backlogging costs is formulated, and the Kuhn-Tucker conditions formulated and solved. Next, an optimization model that includes the lost sales costs is formulated and solved. Finally, some computational results that highlight the advantages of the proposed approach are given and conclusions offered.

Definition of Variables and Parameters

The definitions and notation used here are mostly the same as those used in previous works by the authors.[17,21] As depicted in Figure 1, a supply chain system that converts raw materials into final products through multiple physicochemical processing steps and subsequently transports them to customers is composed of a set of currency storage units (R), a set of material storage units (J), a set of semicontinuous production processes (I), and a set of transportation processes (L). Production process $i \in I$ conducts multiple tasks

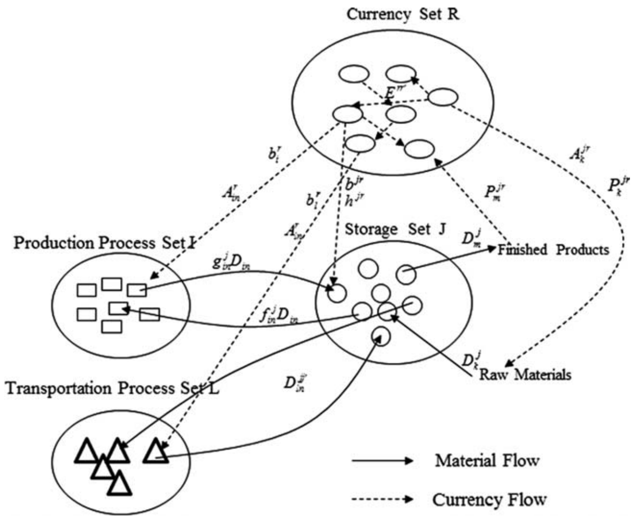

Figure 1. Structure of generalized batch-storage network.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

in sequence, where $n \in N(i)$ is the task index. Tasks are executed in the sequence $n=1, 2,\ldots, |N(i)|$, and this sequence is repeated periodically. If only one task is involved in each process, the results of this study reduce to the case of a batch process.[21] The production process is fed from feedstock storage units $j \in J$ and produces multiple products. Each task requires different feedstock materials and products. The composition of feedstock j in task n in a production process i is f^j_{in}, and the composition of product j from task n in production process i is g^j_{in}, where $\sum_{j=1}^{|J|} f^j_{in} = \sum_{j=1}^{|J|} g^j_{in} = 1$. Note that unlike the assumptions used in our previous work,[17] the feedstock composition and product yield are assumed to be constant in this formulation. B_{in} and ω_{in} are the batch size and duration, respectively, of task n in production process i. Each production process has a common production cycle ω_i and startup time ${}^{\backprime}t_i$. The ratio of ω_{in} to ω_i is denoted by the cycle time ratio y_{in}, that is $\omega_{in} \equiv y_{in}\omega_i$ where $\sum_{n=1}^{|N(i)|} y_{in}=1$ and $0 \le y_{in} \le 1$. $D_{in} \equiv \frac{B_{in}}{\omega_i}$ is the average material flow rate of task n in process i. $D_i \equiv \sum_{n=1}^{|N(i)|} D_{in}$ is the average material flow rate of process i. Thus, by definition $y_{in} = \frac{D_{in}}{D_i}$ holds. The feeding startup time for task n in process i is defined as ${}^{\backprime}t_{in}$, and the discharging startup time of task n in production process i is defined as t'_{in}. Under these definitions, the following timing relationships hold

$$ {}^{\backprime}t_{in} = {}^{\backprime}t_i + \omega_i \sum_{n'=1}^{n-1} y_{in'}, \quad t'_{in} = {}^{\backprime}t_{in} + \Delta t_{in} = {}^{\backprime}t_i + \omega_i \sum_{n'=1}^{n-1} y_{in'} + \Delta t_{in} \quad \forall i,n, \tag{1} $$

where Δt_{in} is the difference between the feeding and discharging startup times of task n in production process i. ${}^{\backprime}x_{in}$ and x'_{in} are defined as the feeding and discharging storage operation times divided by the cycle time ω_i, and these parameters are called the storage operation time fractions (SOTFs). The definitions of ${}^{\backprime}x_{in}$ and x'_{in} differ from those presented in our previous study,[17] where ω_{in} was used instead of ω_i. For semicontinuous processes ${}^{\backprime}x_{in} = x'_{in} = y_{in}$, whereas ${}^{\backprime}x_{in} = x'_{in} = 0$ for batch processes. Consider a multiparcel transportation process $l \in L$, as shown in Figure 3 of Ref. 21. Each transportation process moves multiple parcels of materials of undetermined composition from storage units in a source plant to storage units in a destination plant. The average material flow rate from storage $j \in J$ to storage $j' \in J$ via parcel $n \in N(l)$ in a transportation process $l \in L$ is denoted $D^{jj'}_{ln}$, where storage units j and j' store the same material. Note that the concept of a parcel is the same as that of a task in a production process, and, therefore, both use the same index. The aforementioned definitions for production processes apply to parcel n in transportation process l, and the corresponding notations are B_{ln}, ω_l, ${}^{\backprime}x_{ln}\,(\text{or}\; x'_{ln})$ and ${}^{\backprime}t_{ln}\,(\text{or}\; t'_{ln})$. Note that $\sum_{j=1}^{|J|}\sum_{j' \ne j}^{|J|} D^{jj'}_{ln} = \frac{B_{ln}}{\omega_l} \equiv D_{ln}$. Equations similar to Eq. 1 hold for the transportation processes

$$ {}^{\backprime}t_{ln} = {}^{\backprime}t_l + \omega_l\, {}^{\backprime}y_{ln}, \qquad t'_{ln} = {}^{\backprime}t_l + \omega_l y'_{ln} + \Delta t_{ln} \qquad \forall l, n, \tag{2} $$

where ${}^{\backprime}y_{ln}$ and y'_{ln} define the loading and unloading sequence in a linear function of the cycle time. $\Delta t_{ln}(.)$ is usually constant and can be larger than the cycle time. The overall material balance around a storage unit j gives

$$ \sum_{i=1}^{|I|}\sum_{n=1}^{|N(i)|} g^j_{in} D_{in} + \sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}\sum_{j' \ne j}^{|J|} D^{j'j}_{ln} + \sum_{k=1}^{|K(j)|} D^j_k = \sum_{i=1}^{|I|}\sum_{n=1}^{|N(i)|} f^j_{in} D_{in} + \sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}\sum_{j' \ne j}^{|J|} D^{jj'}_{ln} + \sum_{m=1}^{|M(j)|} D^j_m \qquad \forall j \tag{3} $$

where D^j_k is the average flow rate of material j purchased from supplier $k \in K(j)$, and D^j_m is the average flow rate of the demand for material j by customer $m \in M(j)$. The incoming material flows of purchased feedstock materials are defined by the average flow rate D^j_k, cycle time ω^j_k, startup time t^j_k, and SOTF x^j_k, where the batch size is $B^j_k = D^j_k \omega^j_k$. The same definition holds for a finished product demand flow, such as the average flow rate D^j_m, batch size B^j_m, cycle time ω^j_m, startup time t^j_k, and SOTF x^j_m where $B^j_m = D^j_m \omega^j_m$.

The flow of multiple tasks can be decomposed into multiple flows of a periodic single task. Then, using the machinery of the PSW model with respect to the flow of each task and summing the results of the single-task representations, we can obtain a representation of multiple tasks. The inventory holdup $V^{\,j}(t)$ of storage unit j is represented by the PSW model as in the author's previous work.[17] A storage unit is connected to the incoming flows from suppliers and processes, and the outgoing flows to consumers and processes. The integration of each PSW flow from the initial time to the current time equals the batch size multiplied by the integer part of the time interval over the cycle time, plus an additional term less than or equal to the batch size. That is, the integration of each flow takes the forms

$$ PSW(t; D, \omega, t', x) = D\omega \left[\text{int}\left[\frac{t-t'}{\omega}\right] + \min\left\{1, \frac{1}{x} res\left[\frac{t-t'}{\omega}\right]\right\}\right], \tag{4} $$

or

$$ PSW'(t; B, \omega, t', x) = B \left[\text{int}\left[\frac{t-t'}{\omega}\right] + \min\left\{1, \frac{1}{x} res\left[\frac{t-t'}{\omega}\right]\right\},\right] \tag{5} $$

where D is the average flow rate, B is the batch size, ω is the cycle time, t' is the startup time, x is the SOTF, t is the current time, int[z] is the greatest integer less than or equal to z, and res[z]=z-int[z]. Note that $D=\frac{B}{\omega}$. The inventory holdup function $V^j(t)$ for a storage unit j can be calculated by adding all incoming flows and subtracting all outgoing flows from the initial inventory

$$\begin{aligned} V^j(t)=&V^j(0)+\sum_{k=1}^{|K(j)|} PSW\left(t; D_k^j, \omega_k^j, t_k^j, x_k^j\right) \\ &+\sum_{i=1}^{|I|}\sum_{n=1}^{N(i)} PSW\left(t; g_{in}^j D_{in}, \omega_i, t'_{in}, x'_{in}\right) \\ &-\sum_{i=1}^{|I|}\sum_{n=1}^{N(i)} PSW\left(t; f_{in}^j D_{in}, \omega_i, `t_{in}, `x_{in}\right) \\ &-\sum_{m=1}^{|M(j)|} PSW\left(t; D_m^j, \omega_m^j, t_m^j, x_m^j\right) \\ &-\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}\sum_{j'\neq j}^{|J|} PSW\left(t; D_{ln}^{jj'}, \omega_l, `t_{ln}, `x_{ln}\right) \\ &+\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}\sum_{j'\neq j}^{|J|} PSW\left(t; D_{ln}^{j'j}, \omega_l, t'_{ln}, x'_{ln}\right) \qquad \forall j, \end{aligned} \tag{6}$$

where $V^j(0)$ is the initial inventory holdup. The functions $PSW(.)$ and $PSW'(.)$ have the basic functional form $f(z)=\text{int}[z]+\min\left\{1, \frac{res[z]}{z_1}\right\}$. This function has a lower bound z, an upper bound $z+1-z_1$, and an average $z+0.5(1-z_1)$, which are evident from a graphical representation of $f(z)$.[12] Table 1 of Ref. 20 lists the expressions for the average and the upper and lower bounds of the PSW functions, where $\underline{\underline{PSW}} \leq PSW \leq \overline{\overline{PSW}}, \underline{\underline{PSW'}} \leq PSW' \leq \overline{\overline{PSW'}}, \overline{PSW}=0.5 \times \left(\overline{\overline{PSW}}+\underline{\underline{PSW}}\right)$, and $\overline{PSW'}=0.5\left(\overline{\overline{PSW'}}+\underline{\underline{PSW'}}\right)$. The upper bound of the inventory holdup, the lower bound of the inventory holdup, and the average inventory holdup can be calculated using these properties. The upper bound of the inventory will be used to compute the capital costs of storage units. The lower bound of the inventory will be used as an optimization constraint. The average inventory level will be used to compute the inventory holding costs. The upper/lower bounds and the average level of the inventory holdup $\overline{\overline{V^j}}$, $\underline{\underline{V^j}}$ and $\overline{V^j}$, respectively, are given in Appendix C posted online at http://myweb.pknu.ac.kr/gbyi/.

Costs are paid in multiple currencies $r \in R$. The purchasing setup cost of the raw material j, paid in currency r, is denoted A_k^{jr} (currency/order): the setup cost of the task n in the production process i, paid in currency r, is denoted A_{in}^r (currency/batch), and the setup cost of the parcel n in the transportation process l, paid in currency r, is denoted A_{ln}^r (currency/batch). Note that all material flows are measured volumetrically for convenience. The annual inventory holding cost for material storage j, paid in currency r (currency/L/year), is further segregated into the inventory operating cost (h^{jr}), and the opportunity cost of inventory holding (γ^{jr}). The operating cost accompanies the actual currency flow, but the opportunity cost does not. The operating cost is involved both as a constraint and as an objective function, whereas the opportunity cost is involved only as an objective function. To obtain an analytical solution to this optimization problem, the capital cost is assumed to be proportional to the processing capacity. Suppose that b_k^{jr} (currency/L/year) is the annual capital cost per unit capacity of the purchasing facility for the raw material j, paid in currency r, b_i^r (currency/L/year) is the annual capital cost per unit capacity of production process i, paid in currency r, b_l^r (currency/L/year) is the annual capital cost per unit capacity of transportation process l, paid in currency r, b_m^{jr} (currency/L/year) is the annual capital cost per unit capacity of the sales facility for the finished product j, paid in currency r, and b^{jr} (currency/L/year) is the annual capital cost per unit capacity of storage j, paid in currency r. In addition, assume that the raw material cost is proportional to the quantity, and the purchase price of the raw material j from supplier k, paid in currency r, is P_k^{jr} (currency/L). The sales price of the finished product j to the consumer m, paid in currency r, is P_m^{jr} (currency/L). The annualized cost of the initial inventory preparation is defined as β^{jr} (currency/L/year).

Suppose that there exist currency storage units of the type shown in Figure 4 of Ref. 20 that, through financial transactions, operate on a supply chain consisting of the production process set I, transportation process set L, and material storage unit set J, as depicted in Figure 1. For simplicity, we ignore the effects of temporary financial investments, bank loans, sales tax, and labor costs, which were treated in detail in a previous study.[20] The currency flows entering the currency storage r are CF1 and CF2 in Ref. 21. The currency flows leaving the currency storage r are CF2–CF16 in Ref. 21 and CF17.

(CF17) Annualized initial inventory costs of material storage j proportional to the initial inventory volume β^{jr} (currency/L/year).

CF17 is closely related to the shortage cost. The currency is transferred between currency storage units, (CF2, CF4, and CF8). The average currency flow rate is denoted $E^{r'r}$ (currency/year), where the currency transfer cycle time is $\omega^{r'r}$, the currency transfer startup time is $t^{r'r}$, the currency is transferred from the currency storage r' to the currency storage r, and an exchange rate $\chi^{r'r}$ applies. The SOTF for a currency flow is set to zero without loss of generality. The corresponding transfer setup costs $A^{r'r}$ (currency/transaction) are paid from the sending currency storage r'. For materials transported from storage $j' \in J(r')$ to storage $j \in J(r)$ with an average flow rate of $\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|} D_{ln}^{j'j}$, the purchasing costs will be transferred from the currency storage $r \in R(j)$ to the currency storage $r' \in R(j')$, based on the transfer price $P_{j'j}^{rr'}$, where $J(r)$ is the subset of material storage associated with the currency storage r, and $R(j)$ is the subset of currency storage associated with material storage j. The currency transfer rate is

$$E^{rr'}=\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|} P_{j'j}^{rr'} D_{ln}^{j'j}, \tag{7}$$

Note that the transfer price $P_{j'j}^{rr'}=0$, if $r=r', j=j'$, or $j \notin J(r)$, $j' \notin J(r')$, $r \notin R(j)$ or $r' \notin R(j')$.

In addition, we assume that the setup cost transactions of CF5–CF8 and the inventory operating costs of CF9 are paid on a *pro rata* basis as a function of the extent of material processing or the volume of the financial transactions. The currency flows associated with the inventory operating costs are proportional to the inventory level.

The average currency flow rate for customs duties on the material moved from the material storage j' to the material storage j, transported by parcel n via transportation process l and paid to the nation that uses currency r, is $\pi_{ln}^{j'jr} D_{ln}^{j'j}$, where $\pi_{ln}^{j'jr}$ (currency/L) is the customs duty rate. The variable operating cost, which is the production process operating cost proportional to the average material flow rate through the process, can be treated in the same manner as the customs duty with the variable operating cost rate π_i^r (currency/L). Each currency flow in the PSW model is represented by a function of the batch size (or average flow rate), cycle time, startup time, and SOTF in the same way as the material flows are represented (with appropriate super- or subscripts), as shown in Table 1 in Ref. 21.

The startup times of currency flows of the profit after taxes (benefit to stockholder), and the annualized capital investments are assumed to be zero. The currency flows of the profit after taxes and the annualized capital investments are assumed to be continuous, and the SOTFs for these currency flows are set to one. Invoking these assumptions simplifies the currency flows of the profit after taxes and the annualized capital investments given in the following equation. Table 1 in Ref. 21 lists the functional forms of CF1–CF16, obtained using Eqs. 4 and 5 with the defined variables and parameters. Define $C^r(0)$ as the initial currency inventory and $C^r(t)$ as the currency inventory at time t. Then the currency inventory at time t is calculated by adding the incoming flows (CF1– CF2) to the initial currency inventory and subtracting the outgoing flows (CF3– CF17)

$$\begin{aligned}
C^r(t) = {} & C^r(0) + \sum_{j=1}^{|J|} \sum_{m=1}^{|M(j)|} P_m^{jr} PSW\left(t; D_m^j, \omega_m^j, t_m^j + \Delta t_m^j, x_m^j\right) - \sum_{j \in \{D_m^j\}^+}^{|J|} \sum_{m=1}^{|M(j)|} PSW'\left(t; A_m^{jr}, \omega_m^j, t_m^j, x_m^j\right) \\
& - \sum_{j=1}^{|J|} \sum_{k=1}^{|K(j)|} P_k^{jr} PSW\left(t; D_k^j, \omega_k^j, t_k^j + \Delta t_k^j, x_k^j\right) - \sum_{j \in \{D_k^j\}^+}^{|J|} \sum_{k=1}^{|K(j)|} PSW'\left(t; A_k^{jr}, \omega_k^j, t_k^j, x_k^j\right) \\
& - \sum_{i \in \{D_{in}\}^+}^{|I|} \sum_{n=1}^{|N(i)|} PSW'\left(t; A_{in}^r, \omega_i, `t_{ln}, `x_{in}\right) - \sum_{j=1}^{|J|} h^{jr} \int_0^t V^j(t)\,dt - E_\Sigma^r t - E_T^r t \\
& + \sum_{r' \neq r}^{|R|} \chi^{r'r} PSW\left(t; E^{r'r}, \omega^{r'r}, t^{r'r}, 0\right) - \sum_{i=1}^{|I|} PSW\left(t; \pi_i^r D_i, \omega_i, `t_i, `x_i\right) \\
& - \sum_{r' \neq r}^{|R|} PSW\left(t; E^{rr'}, \omega^{rr'}, t^{rr'}, 0\right) - \sum_{r' \in \{E^{rr'}\}^+}^{|R|} PSW'\left(t; A^{rr'}, \omega^{rr'}, t^{rr'}, 0\right) \\
& - \sum_{l \in \{D_{ln}\}^+}^{|L|} \sum_{n=1}^{|N(l)|} PSW'\left(t; A_{ln}^r, \omega_l, `t_{ln}, `x_{ln}\right) - \sum_{j=1}^{|J|} \sum_{j' \neq j}^{|J|} \sum_{l=1}^{|L|} \sum_{n=1}^{|N(l)|} PSW\left(t; \pi_{ln}^{j'jr} D_{ln}^{j'j}, \omega_l, t'_{ln}, x'_{ln}\right) \\
& - \sum_{j=1}^{|J|} \sum_{k=1}^{|K(j)|} b_k^{jr} D_k^j \omega_k^j t - \sum_{i=1}^{|I|} b_i^r D_i \omega_i t - \sum_{l=1}^{|L|} b_l^r D_l \omega_l t - \sum_{j=1}^{|J|} b^{jr} \overline{\overline{V^j}} t - \sum_{j=1}^{|J|} \beta^{jr} V^j(0) t - \sum_{j=1}^{|J|} \sum_{m=1}^{|M(j)|} b_m^{jr} D_m^j \omega_m^j t \qquad \forall r,
\end{aligned} \tag{8}$$

where $\{D_k^j\}^+ \equiv \{j | D_k^j > 0\}$, $\{D_m^j\}^+ \equiv \{j | D_m^j > 0\}$, $\{D_{in}\}^+ \equiv \{i | D_{in} > 0\}$, $\{D_{ln}\}^+ \equiv \{l | D_{ln} > 0\}$, and $\{E^{r'r}\}^+ \equiv \{r \neq r' | E^{r'r} > 0\}$ are the index sets with positive average flow rates. E_Σ^r (currency/year) is the average profit after taxes, and E_T^r (currency/year) is the average tax flow rate, where $(E_\mathrm{T}^r) = \xi^r (E_\Sigma^r + E_\mathrm{T}^r)$ and ξ^r (currency/currency) is the corporate income tax rate paid in currency r. The average flow rates of currency into and out of a currency storage unit satisfy the following currency balance equations

$$\begin{aligned}
E_\Sigma^r + E_\mathrm{T}^r = {} & \sum_{j=1}^{|J|} \sum_{m=1}^{|M(j)|} P_m^{jr} D_m^j - \sum_{j \in \{D_m^j\}^+}^{|J|} \sum_{m=1}^{|M(j)|} \frac{A_m^{jr}}{\omega_m^j} - \sum_{j=1}^{|J|} \sum_{m=1}^{|M(j)|} b_m^{jr} D_m^j \omega_m^j - \sum_{j=1}^{|J|} \sum_{k=1}^{|K(j)|} P_k^{jr} D_k^j - \sum_{j \in \{D_k^j\}^+}^{|J|} \sum_{k=1}^{|K(j)|} \frac{A_k^{jr}}{\omega_k^j} - \sum_{j=1}^{|J|} \sum_{k=1}^{|K(j)|} b_k^{jr} D_k^j \omega_k^j \\
& - \sum_{i \in \{D_i\}^+}^{|I|} \frac{\sum_{n=1}^{|N(i)|} A_{in}^r}{\omega_i} - \sum_{i=1}^{|I|} b_i^r D_i \omega_i - \sum_{i=1}^{|I|} \pi_i^r D_i - \sum_{r' \in \{E^{rr'}\}^+}^{|R|} \frac{A^{rr'}}{\omega^{rr'}} + \sum_{r' \neq r}^{|R|} \chi^{r'r} E^{r'r} - \sum_{r' \neq r}^{|R|} E^{rr'} \\
& - \sum_{j=1}^{|J|} h^{jr} \overline{V^j} - \sum_{j=1}^{|J|} b^{jr} \overline{\overline{V^j}} - \sum_{j=1}^{|J|} \beta^{jr} V^j(0) - \sum_{l \in \{D_l\}^+}^{|L|} \frac{\sum_{n=1}^{|N(l)|} A_{ln}^r}{\omega_l} - \sum_{l=1}^{|L|} b_l^r D_l \omega_l - \sum_{j=1}^{|J|} \sum_{j' \neq j}^{|J|} \sum_{l=1}^{|L|} \sum_{n=1}^{|N(l)|} \pi_{ln}^{j'jr} D_{ln}^{j'j} \qquad \forall r,
\end{aligned} \tag{9}$$

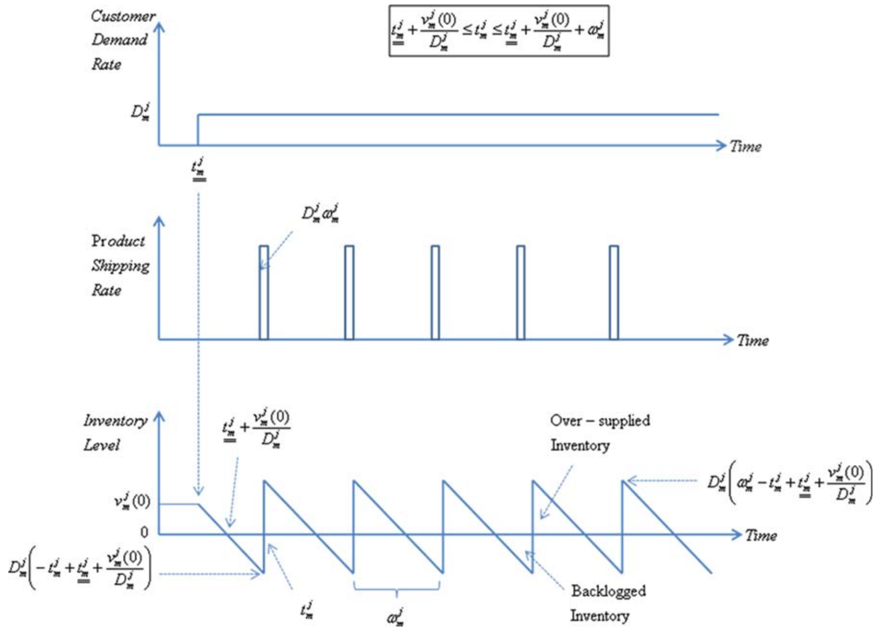

Figure 2. Concept of backlogging cost.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

The average level of the currency inventory $\left(\overline{C^r}\right)$ is calculated by replacing the terms in Eq. 8 with the term $\overline{PSW}(.)$ in Table 1 of Ref. 20. The lower bound of the currency inventory $\left(\underline{\underline{C^r}}\right)$ is calculated by replacing the negative $PSW(.)$ terms in Eq. 8 with the terms $\overline{\overline{PSW}}(.)$, and the positive terms in Eq. 8 are replaced with the $\underline{\underline{PSW}}(.)$ terms. Equation 9 simplifies many terms and the resulting equations C4 and C5 are given in Appendix C posted online at http://myweb.pknu.ac.kr/gbyi/.

Modeling of the backlogging cost

We implement the assumptions of the classical EOQ model with respect to the backlogging cost.[4] The original customer demand is a continuous flow $\left(x^j_m=1\right)$ starting at $\underline{\underline{t}}^j_m$ with an average flow rate D^j_m; however, the product shipping flow pattern is batchwise with cycle time ω^j_m, and instantaneous flow $\left(x^j_m=0\right)$. (The case of $0 < x^j_m < 1$ can be readily handled without theoretical difficulty, but with intensive algebraic details which will not be reported here.) Differences in the customer demand flow patterns and the shipping-to-customer flow patterns result in unavoidable periodic shortages and oversupplies as shown in Figure 2, in which a shortage means the shipping volume is less than the demand volume and oversupply means the opposite. Both an oversupply and a shortage can cause a loss of customer good will (LCGW). Here, two types of hypothetical costs occur: the oversupplied inventory holding cost H^{jr}_m (currency/L/year) and the shortage cost. Note that the oversupplied inventory holding cost differs from the inventory holding cost h^{jr} or γ^{jr}. The shortage costs are classified into two types: the backlogging cost β^{jr}_m (currency/L/year), proportional to the backlogged inventory level, and the lost sales cost α^{jr}_m (currency/L/year), proportional to product volume of the shortage. (Lost sales costs are considered in the next section.) Increasing the initial inventory V^j (0) can reduce the shortage but by incurring additional costs. The fraction of the initial inventory V^j (0) shipped to customer m is denoted $v^j_m(0)$. Then $V^j(0)=\sum_{m=1}^{|M(j)|} v^j_m(0)$. Three inventory profiles should be considered, depending on the range of the shipping startup times t^j_m, as shown in Figures 2 and 3. For $\underline{\underline{t}}^j_m \le t^j_m \le \underline{\underline{t}}^j_m + \frac{v^j_m(0)}{D^j_m}$, as shown in Figure 3, only oversupplied inventory is present, and the oversupplied inventory holding costs are expressed as

$$LCGW\left(t^j_m,\omega^j_m,v^j_m(0)\right)=\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\chi^{r1}H^{jr}_m D^j_m \left(0.5\omega^j_m - t^j_m + \underline{\underline{t}}^j_m + \frac{v^j_m(0)}{D^j_m}\right), \quad (10)$$

For $\underline{\underline{t}}^j_m + \frac{v^j_m(0)}{D^j_m} < t^j_m \le \underline{\underline{t}}^j_m + \frac{v^j_m}{D^j_m} + \omega^j_m$ both oversupplied inventory and backlogged inventory are present, as shown at Figure 2. The sum of the oversupplied inventory holding costs and the backlogged inventory holding costs is

$$LCGW\left(t^j_m,\omega^j_m,v^j_m(0)\right) = \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\frac{\chi^{r1}H^{jr}_m D^j_m\left(t^j_m - \underline{\underline{t}}^j_m - \frac{v^j_m(0)}{D^j_m} - \omega^j_m\right)^2}{2\omega^j_m} + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\frac{\chi^{r1}\beta^{jr}_m D^j_m\left(t^j_m - \underline{\underline{t}}^j_m - \frac{v^j_m(0)}{D^j_m}\right)^2}{2\omega^j_m}, \quad (11)$$

The terms in Eq. 11 are computed as described in Ref. 4. For $\underline{\underline{t}}^j_m + \frac{v^j_m(0)}{D^j_m} + \omega^j_m < t^j_m$ as shown in Figure 3, only backlogged inventory is present, and the backlogged inventory holding costs are

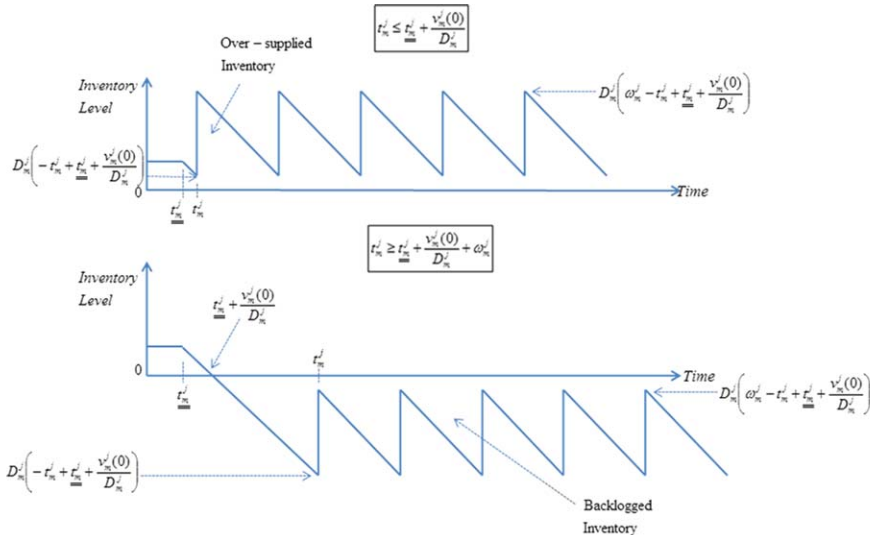

Figure 3. Two other cases of backlogging cost.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

$$LCGW\left(t^j_m, \omega^j_m, v^j_m(0)\right) = \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} \chi^{r1}\beta^{jr}_m D^j_m \times \left(t^j_m - \underline{\underline{t^j_m}} - \frac{v^j_m(0)}{D^j_m} - 0.5\omega^j_m\right), \quad (12)$$

Suppose η^r (currency/currency/year) is the rate of the opportunity costs for the currency inventory (interest rate). The optimization procedure seeks to minimize the opportunity costs of the currency/material inventories minus the profit after taxes, plus the LCGW expressed in the numeraire currency ($r = 1$)

$$Minimize\ \ TC = \sum_{r=1}^{|R|}\chi^{r1}\eta^r\overline{C}^r + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\gamma^{jr}\overline{V^j} - \sum_{r=1}^{|R|}\chi^{r1}E^r_\Sigma + LCGW\left(t^j_m, \omega^j_m, v^j_m(0)\right), \quad (13)$$

Equation 13 can be rewritten using Eq. 9, and the resulting equation Eq. C6 is given in Appendix C posted online at http://myweb.pknu.ac.kr/gbyi/. The optimization problem entails:

(P1) minimizing the objective function in Eq. 13 and one of Eqs. 10, 11 or 12, under the constraints $\underline{\underline{V^j}} \geq 0$, $\underline{\underline{C^r}} \geq 0$, $V^j(0) \geq 0$, $t^j_k \geq \underline{\underline{t^j_k}}$, $\grave{t}_i \leq \grave{t}_i \leq \overline{\overline{t_i}}$, $\grave{t}_l \leq \grave{t}_l \leq \overline{\overline{t_l}}$ and equations given in Eqs. C1–C5 and 9 with respect to the design variables ω^j_k, ω_{in}, ω_{ln}, ω^j_m, $\omega^{rr'}$, t^j_k, $\grave{t}_i$, $\grave{t}_l$, t^j_m, $t^{rr'}$ and $v^j_m(0)$.

Note that the startup times $\grave{t}_{in}$ t'_{in} $\grave{t}_{ln}$ and t'_{ln} are converted into $\grave{t}_i$ and $\grave{t}_l$ via Eqs. 1 and 2, respectively. Note also that ω^j_m and t^j_m are variables here, whereas all previous studies treated them as known parameters. The solution to the Kuhn-Tucker conditions for the optimization problem (P1) can be obtained in analytical form by means of algebraic manipulations summarized in Appendix C posted online at http://myweb.pknu.ac.kr/gbyi/. The optimal cycle times are

$$^*\omega^j_k = \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A^{jr}_k\right)}{D^j_k\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi^{jr}_k\right)}} \quad \forall j,k, \quad (14)$$

where

$$\Psi^{jr}_k \equiv (1-\xi^r)b^{jr}_k + \left[0.5\eta^r P^{jr}_k + \theta^{jr}\right]\left(1-x^j_k\right) \quad \forall j,k, \quad (15)$$

with

$$\theta^{jr} \equiv \frac{(1-\xi^r)h^{jr} + \gamma^{jr}}{2} + (1-\xi^r)b^{jr} \quad \forall j,r, \quad (16)$$

$$^*\omega_i = \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)\sum_{n=1}^{|N(i)|}A^r_{in}\right)}{\left(\sum_{r=1}^{|R|}\sum_{n=1}^{|N(i)|}\chi^{r1}\Psi^r_{in}D_{in}\right)}} \quad \forall i \quad (17)$$

$$\Psi^r_{in} \equiv (1-\xi^r)b^r_i + (1-\grave{x}_{in})\sum_{j=1}^{|J|}\theta^{jr}f^j_{in} + (1-x'_{in})\sum_{j=1}^{|J|}\theta^{jr}g^j_{in} + (1-\grave{x}_i)\sum_{j=1}^{|J|}0.5\eta^r\pi^r_i - \sum_{j=1}^{|J|}\Omega(j)\left[\left(f^j_{in} - g^j_{in}\right)\left(\sum_{n'=1}^{n-1}y_{in'}\right) - (1-\grave{x}_{in})f^j_{in} - g^j_{in}\frac{\partial\Delta t_{in}}{\partial\omega_i}\right] \quad \forall i,r,n \quad (18)$$

and where $\Omega(j)$ is defined in Tables 1–4. T1-T4

$$^*\omega_l = \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)\sum_{n=1}^{|N(l)|}A^r_{ln}\right)}{\left(\sum_{r=1}^{|R|}\chi^{r1}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N(l)|}\Psi^{jj'r}_{ln}D^{jj'}_{ln}\right)}} \quad \forall l, \quad (19)$$

$$\Psi^{jj'r}_{ln} \equiv (1-\xi^r)b^r_l + (1-\grave{x}_{ln})\theta^{jr} + (1-x'_{ln})\left(\theta^{j'r} + 0.5\eta^r\pi^{jj'r}_{ln}\right) + \Omega(j)\left[(1-\grave{x}_{ln}) - \grave{y}_{ln}\right] + \Omega(j')y'_{ln} \quad \forall l,n,j,j',r, \quad (20)$$

$$^*\omega^j_m = \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A^{jr}_m\right)}{D^j_m\Psi^j_m}} \quad \forall j,m \quad (21)$$

$$\Psi^j_m \equiv \sum_{r=1}^{|R|}\chi^{r1}\left[\left(\theta^{jr} + 0.5\eta^r P^{jr}_m\right) + (1-\xi^r)b^{jr}_m\right] + \Delta\Psi^j_m \quad \forall j,m \quad (22)$$

where $\Delta\Psi^j_m$ is defined in Table 1–4

Table 1. Solution 1 ~ 11 for Backlogging Case

No.	Parameter Restriction	$V^j(0)$	Process Startup Times	$\Omega(j)$
1		0	$t_k^j > \underline{\underline{t_k^j}}$ $\underline{\underline{\grave{t}_i}} < \grave{t}_i < \overline{\overline{\grave{t}_i}}$ $\underline{\underline{\grave{t}_l}} < \grave{t}_l < \overline{\overline{\grave{t}_l}}$	0
2		$V^j(0) > 0$	$t_k^j = \underline{\underline{t_k^j}}$ $\grave{t}_i = \overline{\overline{\grave{t}_i}}$ or $\grave{t}_i = \underline{\underline{\grave{t}_i}}$ $\grave{t}_l = \overline{\overline{\grave{t}_l}}$ or $\grave{t}_l = \underline{\underline{\grave{t}_l}}$	$0.5(1-\xi^r)\beta^{jr}$
3		0	$t_k^j = \underline{\underline{t_k^j}}$ $\grave{t}_i = \overline{\overline{\grave{t}_i}}$ or $\grave{t}_i = \underline{\underline{\grave{t}_i}}$ $\grave{t}_l = \overline{\overline{\grave{t}_l}}$ or $\grave{t}_l = \underline{\underline{\grave{t}_l}}$	$\Omega(j) < 0.5(1-\xi^r)\beta^{jr}$
4		0	$t_k^j > \underline{\underline{t_k^j}}$ $\underline{\underline{\grave{t}_i}} < \grave{t}_i < \overline{\overline{\grave{t}_i}}$ $\underline{\underline{\grave{t}_l}} < \grave{t}_l < \overline{\overline{\grave{t}_l}}$	0
5	$0.5(1-\xi^r)\beta^{jr} \le \beta_m^{jr}$	$V^j(0) > 0$	$t_k^j = \underline{\underline{t_k^j}}$ $\grave{t}_i = \overline{\overline{\grave{t}_i}}$ or $\grave{t}_i = \underline{\underline{\grave{t}_i}}$ $\grave{t}_l = \overline{\overline{\grave{t}_l}}$ or $\grave{t}_l = \underline{\underline{\grave{t}_l}}$	$0.5(1-\xi^r)\beta^{jr}$
6	$0.5(1-\xi^r)\beta^{jr} \ge \beta_m^{jr}$	$V^j(0) > 0$	$t_k^j = \underline{\underline{t_k^j}}$ $\grave{t}_i = \overline{\overline{\grave{t}_i}}$ or $\grave{t}_i = \underline{\underline{\grave{t}_i}}$ $\grave{t}_l = \overline{\overline{\grave{t}_l}}$ or $\grave{t}_l = \underline{\underline{\grave{t}_l}}$	$0.5(1-\xi^r)\beta^{jr}$
7		0	$t_k^j = \underline{\underline{t_k^j}}$ $\grave{t}_i = \overline{\overline{\grave{t}_i}}$ or $\grave{t}_i = \underline{\underline{\grave{t}_i}}$ $\grave{t}_l = \overline{\overline{\grave{t}_l}}$ or $\grave{t}_l = \underline{\underline{\grave{t}_l}}$	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\Omega(j) <$ $\min\left\{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}, \sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5(1-\xi^r)\beta^{jr}\right\}$
8		0	$t_k^j = \underline{\underline{t_k^j}}$ $\grave{t}_i = \overline{\overline{\grave{t}_i}}$ or $\grave{t}_i = \underline{\underline{\grave{t}_i}}$ $\grave{t}_l = \overline{\overline{\grave{t}_l}}$ or $\grave{t}_l = \underline{\underline{\grave{t}_l}}$	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5(1-\xi^r)\beta^{jr} >$ $\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\Omega(j) \ge \sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}$
9		0	$t_k^j > \underline{\underline{t_k^j}}$ $\underline{\underline{\grave{t}_i}} < \grave{t}_i < \overline{\overline{\grave{t}_i}}$ $\underline{\underline{\grave{t}_l}} < \grave{t}_l < \overline{\overline{\grave{t}_l}}$	0
10	$0.5(1-\xi^r)\beta^{jr} \ge \beta_{\underset{\smile}{m}}^{jr}$	0	$t_k^j = \underline{\underline{t_k^j}}$ $\grave{t}_i = \overline{\overline{\grave{t}_i}}$ or $\grave{t}_i = \underline{\underline{\grave{t}_i}}$ $\grave{t}_l = \overline{\overline{\grave{t}_l}}$ or $\grave{t}_l = \underline{\underline{\grave{t}_l}}$	$\beta_{\underset{\smile}{m}}^{jr}$
11		0	$t_k^j = \underline{\underline{t_k^j}}$ $\grave{t}_i = \overline{\overline{\grave{t}_i}}$ or $\grave{t}_i = \underline{\underline{\grave{t}_i}}$ $\grave{t}_l = \overline{\overline{\grave{t}_l}}$ or $\grave{t}_l = \underline{\underline{\grave{t}_l}}$	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\Omega(j) <$ $\min\left\{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}, \sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5(1-\xi^r)\beta^{jr}\right\}$

Table 2. Solution 1 ~ 11 for Backlogging Case Extended

No.	$\Delta\Psi_m^j$	t_m^j
1	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5H_m^{jr}$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}$
2	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left[0.5H_m^{jr}+0.5(1-\xi^r)\beta^{jr}\right]$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}$
3	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left[0.5H_m^{jr}+\Omega(j)\right]$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}$
4	$\frac{0.5\left(\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}\right)\left(\sum_{r=1}^{\lvert R\rvert}\chi^{r1}H_m^{jr}\right)}{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(\beta_m^{jr}+H_m^{jr}\right)}$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}+\frac{\left(\sum_{r=1}^{\lvert R\rvert}\chi^{r1}H_m^{jr}\right)}{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(\beta_m^{jr}+H_m^{jr}\right)}\left({}^{*}\omega_m^j\right)$
5	$\frac{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}(1-\xi^r)\beta^{jr}\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}+\sum_{r=1}^{\lvert R\rvert}\chi^{r1}H_m^{jr}\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}-\left\{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5(1-\xi^r)\beta^{jr}\right\}^2}{2\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(\beta_m^{jr}+H_m^{jr}\right)}$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}+\frac{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(0.5(1-\xi^r)\beta^{jr}+H_m^{jr}\right)}{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(\beta_m^{jr}+H_m^{jr}\right)}\left({}^{*}\omega_m^j\right)$
6	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5\beta_m^{jr}$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}+\left({}^{*}\omega_m^j\right)$
7	$\frac{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\Omega(j)\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}+0.5\sum_{r=1}^{\lvert R\rvert}\chi^{r1}H_m^{jr}\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}-0.5\left\{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\Omega(j)\right\}^2}{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(\beta_m^{jr}+H_m^{jr}\right)}$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}+\frac{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(\Omega(j)+H_m^{jr}\right)}{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(\beta_m^{jr}+H_m^{jr}\right)}\left({}^{*}\omega_m^j\right)$
8	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5\beta_m^{jr}$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}+\left({}^{*}\omega_m^j\right)$
9	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5\beta_m^{jr}$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}+\left({}^{*}\omega_m^j\right)$
10	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5\beta_m^{jr}$	$t^j_{m\neq\underset{\smile}{m}}=\underline{\underline{t^j_{m\neq\underset{\smile}{m}}}}+\frac{v^j_{m\neq\underset{\smile}{m}}(0)}{D^j_{m\neq\underset{\smile}{m}}}+\left({}^{*}\omega^j_{m\neq\underset{\smile}{m}}\right)$
		$t^j_{\underset{\smile}{m}}>\underline{\underline{t^j_{\underset{\smile}{m}}}}+\frac{v^j_{\underset{\smile}{m}}(0)}{D^j_{\underset{\smile}{m}}}+\left({}^{*}\omega^j_{\underset{\smile}{m}}\right)$
11	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5\beta_m^{jr}$	$\underline{\underline{t_m^j}}+\frac{v_m^j(0)}{D_m^j}+\left({}^{*}\omega_m^j\right)$

Table 3. Solution 12 ~ 14 for Lost Sales Case

No.	Parameter Restriction	$V^j(0)$	Process Startup Times	$\Omega(j)$
12	$0.5(1-\xi^r)\beta^{jr}<\alpha^{jr}_{\underset{\smile}{m}}$	$V^j(0)>0$	$t_k^j=\underline{\underline{t_k^j}}$ ${}^{\backprime}t_i=\overline{\overline{{}^{\backprime}t_i}}$ or ${}^{\backprime}t_i=\underline{\underline{{}^{\backprime}t_i}}$ ${}^{\backprime}t_l=\overline{\overline{{}^{\backprime}t_l}}$ or ${}^{\backprime}t_l=\underline{\underline{{}^{\backprime}t_l}}$	$0.5(1-\xi^r)\beta^{jr}$
13	$0.5(1-\xi^r)\beta^{jr}\geq\alpha^{jr}_{\underset{\smile}{m}}$	0	$t_k^j=\underline{\underline{t_k^j}}$ ${}^{\backprime}t_i=\overline{\overline{{}^{\backprime}t_i}}$ or ${}^{\backprime}t_i=\underline{\underline{{}^{\backprime}t_i}}$ ${}^{\backprime}t_l=\overline{\overline{{}^{\backprime}t_l}}$ or ${}^{\backprime}t_l=\underline{\underline{{}^{\backprime}t_l}}$	$\alpha^{jr}_{\underset{\smile}{m}}$
14		0	$t_k^j=\underline{\underline{t_k^j}}$ ${}^{\backprime}t_i=\overline{\overline{{}^{\backprime}t_i}}$ or ${}^{\backprime}t_i=\underline{\underline{{}^{\backprime}t_i}}$ ${}^{\backprime}t_l=\overline{\overline{{}^{\backprime}t_l}}$ or ${}^{\backprime}t_l=\underline{\underline{{}^{\backprime}t_l}}$	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\Omega(j)<$ $\min\left\{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\alpha_m^{jr},\sum_{r=1}^{\lvert R\rvert}\chi^{r1}0.5(1-\xi^r)\beta^{jr}\right\}$

$$
{}^{*}\omega^{rr'}=\sqrt{\frac{(1-\xi^{r})A^{rr'}}{E^{rr'}\Psi^{rr'}}} \qquad \forall r,r', \tag{23}
$$

where

$$
\Psi^{rr'}\equiv 0.5\left(\left(\frac{\chi^{r'1}}{\chi^{r1}}\right)\eta^{r'}\chi^{rr'}+\eta^{r}\right) \qquad \forall r,r', \tag{24}
$$

$\underline{\underline{V}}^{j}=0$ with Eq. C2 and with Eq. C5 give

$$
\begin{aligned}
&\sum_{k=1}^{|K(j)|}D_k^j t_k^j+\sum_{i=1}^{|I|}\sum_{n=1}^{|N(i)|}\left(g_{in}^j-f_{in}^j\right)D_{in}`t_i+\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}\sum_{j'\neq j}^{|J|}\left(D_{ln}^{j'j}-D_{ln}^{jj'}\right)`t_l\\
&-\sum_{m=1}^{|M(j)|}D_m^j t_m^j=V^j(0)-\sum_{i=1}^{|I|}\sum_{n=1}^{|N(i)|}g_{in}^j D_{in}\Delta t_{in}\\
&-\sum_{i=1}^{|I|}\sum_{n=1}^{|N(i)|}\left(g_{in}^j-f_{in}^j\right)D_{in}({}^{*}\omega_i)\sum_{n'=1}^{n-1}y_{in'}\\
&-\sum_{i=1}^{|I|}\sum_{n=1}^{|N(i)|}(1-`x_{in})f_{in}^j D_{in}({}^{*}\omega_i)-\sum_{m=1}^{|M(j)|}(1-x_m^j)D_m^j\omega_m^j\\
&-\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}\sum_{j'\neq j}^{|J|}(1-`x_{ln}\text{-}`y_{ln})D_{ln}^{jj'}({}^{*}\omega_l)\\
&-\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}\sum_{j'\neq j}^{|J|}D_{ln}^{j'j}\left[({}^{*}\omega_l)y'_{ln}+\Delta t_{ln}\right] \qquad \forall j,
\end{aligned} \tag{25}
$$

and

$$
\begin{aligned}
&\sum_{r'\neq r}^{|R|}\chi^{r'r}E^{r'r}t^{r'r}+\sum_{r'\neq r}^{|R|}\left[\left({}^{*}\omega^{rr'}\right)E^{rr'}-E^{rr'}t^{rr'}-\frac{A^{rr'}}{\left({}^{*}\omega^{rr'}\right)}\right]\\
&=C^r(0)-\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j\left(t_m^j+\Delta t_m^j\right)\\
&-\sum_{j\in\left\{D_m^j\right\}^+}^{|J|}\sum_{m=1}^{|M(j)|}A_m^{jr}\left[(1-x_m^j)-\frac{t_m^j}{\left({}^{*}\omega_m^j\right)}\right]\\
&-\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}\left[\left(1-x_k^j\right)D_k^j\left({}^{*}\omega_k^j\right)-D_k^j\left(t_k^j+\Delta t_k^j\right)\right]\\
&-\sum_{i\in\left\{D_{in}\right\}^+}^{|I|}\sum_{n=1}^{|N(i)|}A_{in}^r\left[(1-`x_{in})-\frac{`t_i}{({}^{*}\omega_i)}-\sum_{n'=1}^{n-1}y_{in'}\right]\\
&-\sum_{j\in\left\{D_k^j\right\}^+}^{|J|}\sum_{k=1}^{|K(j)|}A_k^{jr}\left[\left(1-x_k^j\right)-\frac{t_k^j}{\left({}^{*}\omega_k^j\right)}\right]\\
&-\sum_{r'\in\left\{E^{rr'}\right\}^+}^{|R|}A^{rr'}-\sum_{i=1}^{|I|}\pi_i^r D_i\left[(1-`x_i)({}^{*}\omega_i)-`t_i\right]\\
&-\sum_{l\in\left\{D_{ln}\right\}^+}^{|L|}\sum_{n=1}^{|N(l)|}A_{ln}^r\left[(1-`x_{ln})-\frac{`t_l}{({}^{*}\omega_l)}-`y_{ln}\right]
\end{aligned} \tag{26}
$$

The optimized objective function is

$$
\begin{aligned}
&{}^{*}TC\left(D_k^j,D_{in},D_{ln}^{jj'}\right)=\\
&2\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}\right)\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_k^{jr}\right)D_k^j}+\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\chi^{r1}(1-\xi^r)P_k^{jr}D_k^j\\
&+2\sum_{i=1}^{|I|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)\sum_{n=1}^{|N(i)|}A_{in}^r\right)\left(\sum_{r=1}^{|R|}\sum_{n=1}^{|N(i)|}\chi^{r1}\Psi_{in}^r D_{in}\right)}+\sum_{r=1}^{|R|}\sum_{i=1}^{|I|}\chi^{r1}(1-\xi^r)\pi_i^r\sum_{n=1}^{|N(i)|}D_{in}\\
&+2\sum_{l=1}^{|L|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)\sum_{n=1}^{|N(l)|}A_{ln}^r\right)\left(\sum_{r=1}^{|R|}\chi^{r1}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{n=1}^{|N(l)|}\Psi_{ln}^{jj'r}D_{ln}^{jj'}\right)}+\sum_{\mathrm{r=1}}^{|\mathrm{R}|}\sum_{\mathrm{j=1}}^{|\mathrm{J}|}\sum_{\mathrm{j'\neq j}}^{|\mathrm{J}|}\sum_{\mathrm{l=1}}^{|\mathrm{L}|}\sum_{\mathrm{n=1}}^{|\mathrm{N(l)}|}\chi^{r1}(1-\xi^r)\pi_{ln}^{jj'r}D_{ln}^{jj'}\\
&+\sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\left(\chi^{r1}(1-\xi^r)-\chi^{r1}\chi^{rr'}\left(1-\xi^{r'}\right)\right)\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{l=1}^{|L|}\sum_{n=1}^{|N|}P_{j'j}^{rr'}D_{ln}^{j'j}+2\sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\chi^{r1}\sqrt{(1-\xi^r)A^{rr'}\Psi^{rr'}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{l=1}^{|L|}\sum_{n=1}^{|N|}P_{j'j}^{rr'}D_{ln}^{j'j}}\\
&+0.5\sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{i\in\left\{D_{in}\right\}^+}^{|I|}\sum_{n=1}^{|N(i)|}A_{in}^r(1-`x_{in})+0.5\sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{j\in\left\{D_k^j\right\}^+}^{|J|}\sum_{k=1}^{|K(j)|}\left(1-x_k^j\right)A_k^{jr}+0.5\sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{\mathrm{l}\in\left\{\mathrm{D_{ln}}\right\}^+}^{|\mathrm{L}|}\sum_{\mathrm{n=1}}^{|\mathrm{N(l)}|}A_{ln}^r(1-`x_{ln})\\
&+0.5\sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{r'\in\left\{E^{rr'}\right\}^+}^{|R|}A^{rr'}+0.5\sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{j\in\left\{D_m^j\right\}^+}^{|J|}\sum_{m=1}^{|K(j)|}(1-x_m^j)A_m^{jr}+2\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_m^{jr}\right)\Psi_m^j D_m^j}\\
&-\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\chi^{r1}(1-\xi^r)P_m^{jr}D_m^j-\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\Omega(j)\left[\sum_{m=1}^{|M(j)|}D_m^j\underline{\underline{t_m^j}}-\sum_{k=1}^{|K(j)|}D_k^j\underline{\underline{t_k^j}}+\sum_{i=1}^{|I|}\sum_{n=1}^{|N(i)|}f_{in}^j D_{in}\overline{\overline{`t_i}}-\sum_{i=1}^{|I|}\sum_{n=1}^{|N(i)|}g_{in}^j D_{in}\underline{\underline{`t_i}}\right]\\
&-\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\Omega(j)\left[\sum_{j'\neq j}^{|J|}\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}D_{ln}^{jj'}\overline{\overline{`t_l}}-\sum_{j'\neq j}^{|J|}\sum_{l=1}^{|L|}\sum_{n=1}^{|N(l)|}D_{ln}^{j'j}\underline{\underline{`t_l}}\right],
\end{aligned} \tag{27}
$$

Table 4. Solution 11 ~ 14 for Lost Sales Case Extended.

No.	$\Delta\Psi_m^j$	t_m^j
12	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left[0.5(1-\xi^r)\beta^{jr}+0.5H_m^{jr}\right]$	$\underline{\underline{t}}_m^j+\frac{v_m^j(0)}{D_m^j}$
13	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left[\alpha_{\breve{m}}^{jr}+0.5H_m^{jr}\right]$	$t^j_{m\neq\breve{m}}=\underline{\underline{t}}^j_{m\neq\breve{m}}+\frac{v^j_{m\neq\breve{m}}(0)}{D^j_{m\neq\breve{m}}}$, $t^j_{\breve{m}}>\underline{\underline{t}}^j_{\breve{m}}+\frac{v^j_{\breve{m}}}{D^j_{\breve{m}}}$
14	$\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left[\Omega(j)+0.5H_m^{jr}\right]$	$\underline{\underline{t}}_m^j+\frac{v_m^j(0)}{D_m^j}$

For $\underline{t}_m^j \le t_m^j \le \underline{\underline{t}}_m^j+\frac{v_m^j(0)}{D_m^j}$, we have Solutions 1–3. For $\underline{\underline{t}}_m^j+\frac{v_m^j(0)}{D_m^j} < t_m^j \le \underline{\underline{t}}_m^j+\frac{v_m^j(0)}{D_m^j}+\omega_m^j$, we have Solutions 4–8. For $\underline{\underline{t}}_m^j+\frac{v_m^j(0)}{D_m^j}+\omega_m^j < t_m^j$, we have Solutions 9–11. Solutions 1–11 are given in Tables 1 and 2 where $\breve{m}=\arg\min_m\left\{\alpha_m^j \text{ or } \beta_m^j\right\}$.

We have 11 solutions for the backlogging cost depending on parametric range, but Solutions 1, 2, 3, 6, 7, 8, 9, and 11 are redundant. If $\beta_m^{jr} >> H_m^{jr}$, Solution 4 collapses to Solution 1, Solution 5 collapses to Solution 2, and Solution 7 collapses to Solution 3. If, $H_m^{jr} >> \beta_m^{jr}$ Solution 4 collapses to Solution 9, and Solution 7 collapses to Solution 11. Solution 10 dominates Solution 6 in objective function value. For $0.5(1-\xi^r)\beta^{jr}=\beta_{\breve{m}}^{jr}$, $V^j(0)>0$ is possible in Solution 10. Note that Eq. 25 determines the remaining freedoms of the variables. $\Omega(j)$ in Solutions 3, 7, 8, and 11 is determined from Eq. 27, which is a highly nonlinear implicit equation. The optimized objective function Eq. 33 is composed of the sum of the square root and linear terms with respect to $\Omega(j)$, and, thus, is a concave function. A global optimum exists, therefore, at the bounds of $\Omega(j)$: $\Omega(j)=0,\ 0.5(1-\xi^r)\beta^{jr}$ or $\beta_{\breve{m}}^{jr}$, which are Solutions 4, 5, and 10. Now, we have three remaining solutions. The optimal solution to the classical EOQ model with a backlogging cost,[4] which did not consider multiple currency and exchange rate can be rewritten in the notations used in this article as

$$\Delta\Psi_m^j=\frac{0.5\left(\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\beta_m^{jr}\right)\left(\sum_{r=1}^{\lvert R\rvert}\chi^{r1}H_m^{jr}\right)}{\sum_{r=1}^{\lvert R\rvert}\chi^{r1}\left(\beta_m^{jr}+H_m^{jr}\right)}\qquad\forall j,m,\qquad(28)$$

Solution 4 corresponds to the classical EOQ model with a backlogging cost (Eq. 28). Many important observations may be made here. The usage of the classical EOQ model with backlogging cost is more restrictive than as is known so far. Solution 4 in Table 1 indicates that process startup times are not binding and still more process freedom exists to reduce shortage although it increases total cost. Solution 4 does not use initial inventory to reduce shortage because of cost increase. Note that the process startup times of Solution 10 has no freedom to adjust to reduce shortage, and, therefore, Solution 10 is more popular in the real world. The lot size computed using Solution 4 is smaller than that predicted in the classical EOQ model with a backlogging cost (Eq. 28), because of the additional terms in Eq. 22. Solution 4 does not influence the optimal value of the production/transportation process cycle times or the average flow rates through the network, although Solutions 5 and 10 do have such an influence. (Additional terms include $\Omega(j)>0$ in Eqs. 18 and 20.) Solution 10 indicates that an individual customer's backlogging cost can strongly influence the whole network structure as well as the lot size of product in shortage, and, therefore, backlogging costs should be managed customer by customer.

Modeling of the lost sales cost

The classical EOQ model that includes lost sales costs assumes that the lost sales occur periodically for an indefinite time, and the average product shipping rate is less than the original customer demand rate. The resulting optimal solution is the one with no shortage, which is a trivial one.[4] We impose a different assumption, that is, lost sales occur once when a product is in shortage and once when a product is ready to deliver, and no other shortages occur without customer demand reduction, as can be seen in Figure 4. The shortage cost is computed by adding the lost sales cost during the shortage period, which is proportional to the shortage volume, and the oversupplied inventory holding cost, which is proportional to half the lot size

$$LCGW\left(t_m^j,\omega_m^j,v_m^j(0)\right)=\sum_{r=1}^{\lvert R\rvert}\sum_{j=1}^{\lvert J\rvert}\sum_{m=1}^{\lvert M(j)\rvert}\chi^{r1}\alpha_m^{jr}D_m^j\left(t_m^j-\underline{\underline{t}}_m^j-\frac{v_m^j(0)}{D_m^j}\right)+0.5\sum_{r=1}^{\lvert R\rvert}\sum_{j=1}^{\lvert J\rvert}\sum_{m=1}^{\lvert M(j)\rvert}\chi^{r1}H_m^{jr}D_m^j\omega_m^{jr},\qquad(29)$$

Three solutions to the Kuhn-Tucker conditions exist, as described in Appendix C posted online at http://myweb.pknu.ac.kr/gbyi/, and are summarized in Tables 3 and 4. Solution 12 is the same as Solution 2, and Solution 14 is the same as Solution 3. The independent solutions to the shortage cost developed in this study are summarized as Solutions 4, 5, 10 and 13.

If both customers associated with backlogging and customers associated with lost sales exist for the same product, all combinations of the aforementioned Solutions 1–11 and Solutions 12– 14 may be obtained. The solutions will differ depending on the choice of the term in the Lagrange multiplier $\Omega(j)$, which is limited to $0\le\Omega(j)\le\min\left\{\alpha_{\breve{m}}^{jr},\beta_{\breve{m}}^{jr},0.5(1-\xi^r)\beta^{jr}\right\}$. We omit the solutions for the combined case here for the purpose of clarity. A key assumption of the classical EOQ with a backlogging cost is that the product shipping flow is batch-wise, whereas customer demand flow is continuous. This assumption is required to compute the oversupplied inventory level and the backlogged inventory level. The product shipping flow and customer demand flow can occur in several combinations, and their solutions will be different. Solutions 10 and 13 assume that the customer waits for the delivery of the product in shortage forever, but this is not realistic if the waiting time goes beyond a certain limit. A customer can switch from a backlog category to a lost sales category depending on the waiting time length. The full spectrum of shortage costs should be examined in the following research.

Simplified network design model

The average flow rates D_k^j, D_{in} and $D_{ln}^{jj'}$ and production sequences were assumed to be known parameters in the aforementioned development; however, they were variables

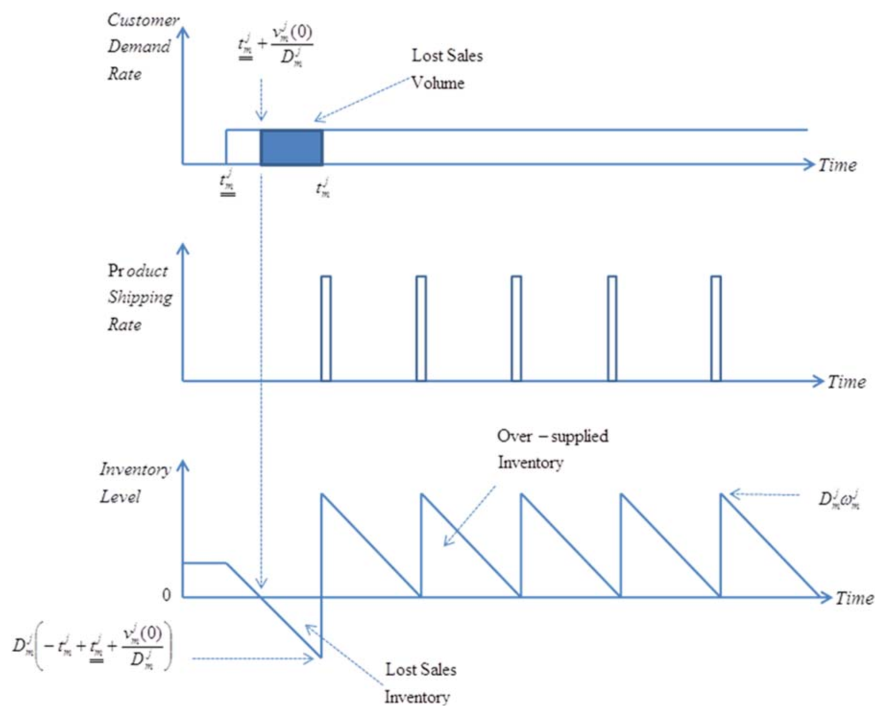

Figure 4. Concept of lost sales cost.

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to be computed prior to the cycle times and startup times. (The global optimality of such a two-level approach was proven previous.)[14] The optimization model used to compute the average flow rates is composed of an objective function (Eq. 27), and a constraint (Eq. 3). The equations used to compute the production sequence are given in Eq. 21 in Ref. 17. The optimization model is highly nonlinear and includes a four-variable term $D_i \grave{x}_{in}\left({}^*\omega_{in}\right)\sum_{n'=1}^{n-1} y_{in'}$, with a large number of binary variables that are required in order to define the production sequence, and, thus, direct solution is beyond the practical limitations of current computational resources. We, therefore, introduce a simplified optimization model. Assume that all products are produced once within a common cycle ($N(i)=J$), and the sequence is arbitrary, say $N=j$. The validity of this assumption is based on the fact that if $\min_n \sum_{r=1}^{|R|} \chi^{r1} A_{in}^r > 0.5 \max_n \sum_{r=1}^{|R|} \chi^{r1} A_{in}^r$, it is optimal to allocate all different products to the tasks in a common cycle.[17] Then, $\grave{x}_{in}=y_{in}=\grave{x}_{ij}=\frac{D_{ij}}{D_i}$ and $D_{in}=D_{ij}$. Assume that only the average production setup cost $A_{in}^r=A_i^r$ is known. Equation 3 is converted to

$$\sum_{i=1}^{|I|} D_{ij}+\sum_{l=1}^{|L|}\sum_{j'\neq j}^{|J|} D_l^{j'j}=\sum_{l=1}^{|L|}\sum_{j'\neq j}^{|J|} D_l^{jj'}+\sum_{m=1}^{|M(j)|} D_m^j \qquad \forall j \tag{30}$$

where a transportation process involves only one parcel, for simplicity. The cycle times are

$${}^*\omega_i=\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)|J|A_i^r\right)}\frac{1}{\sqrt{(\Xi_i)}} \qquad \forall i, \tag{31}$$

where

$$\Xi_i=\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\Psi_{ij}^r D_{ij}=\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)b_i^r D_i +\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\theta^{jr}\left(1-\grave{x}_{ij}\right)\grave{x}_{ij}D_i \qquad \forall i \tag{32}$$

$${}^*\omega_l=\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_l^r\right)}\frac{1}{\sqrt{(\Xi_l)}} \qquad \forall l, \tag{33}$$

and where

$$\Xi_l=\sum_{r=1}^{|R|}\chi^{r1}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\Psi_l^{jj'r}D_l^{jj'} \tag{34}$$

$\underline{\underline{V^j}}\left(\overline{\overline{t_k^j}},\overline{\overline{\grave{t}_i}},\underline{\underline{t_m^j}},\overline{\overline{t'_1}},\underline{\underline{\grave{t}_1}}\right)\geq 0$ in Eq. C2 gives the absolute network conditions with no product shortage, where the upper double bar represents the upper bound and the lower double bar represents the lower bound

$$\begin{aligned}&V^j(0)-\sum_{k=1}^{|K(j)|}D_k^j\overline{\overline{t_k^j}}-\sum_{i=1}^{|I|}D_i\grave{x}_{ij}\left(\overline{\overline{\grave{t}_i}}+\left({}^*\omega_i\right)|_{\Omega(j)=0}+\Delta t_{ij}\right)\\&-\sum_{m=1}^{|M(j)|}(1-x_m^{in})D_m^j\omega_m^j+\sum_{m=1}^{|M(j)|}D_m^j\underline{\underline{t_m^j}}\\&-\sum_{l=1}^{|L|}\sum_{j'\neq j}^{|J|}D_l^{j'j}\overline{\overline{t'_l}}-\sum_{l=1}^{|L|}\sum_{j'\neq j}^{|J|}(1-\grave{x}_l)D_l^{jj'}\left({}^*\omega_l\right)|_{\Omega(j)=0}\\&+\sum_{l=1}^{|L|}\sum_{j'\neq j}^{|J|}D_l^{jj'}\underline{\underline{\grave{t}_l}}\geq 0 \qquad \forall j\end{aligned} \tag{35}$$

where $\sum_{j'=1}^{j-1} y_{ij'}(\leq 1)$ in Eq. 1 is removed to provide tighter constraints. The objective function Eq. 27 is simplified to

$$
\begin{aligned}
{}^{*}TC\left(D_i, D_l^{jj'}\right) = {} & 2\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}\right)\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_k^{jr}\right)D_k^j} + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\chi^{r1}(1-\xi^r)P_k^{jr}D_k^j \\
& + 2\sum_{i=1}^{|I|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)|J|A^r\right)(\Xi_i)} + \sum_{r=1}^{|R|}\sum_{i=1}^{|I|}\chi^{r1}(1-\xi^r)\pi_i^r D_i \\
& + 2\sum_{l=1}^{|L|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_l^r\right)(\Xi_l)} + (1-\xi^r)\sum_{j=1}^{|J|}\chi^{r1}\beta^{jr}V^j(0) \\
& + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{l=1}^{|L|}\chi^{r1}(1-\xi^r)\pi_l^{jj'r}D_l^{jj'} + \sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\left(\chi^{r1}(1-\xi^r)-\chi^{r'1}\chi^{rr'}\left(1-\xi^{r'}\right)\right)\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{l=1}^{|L|}P_{j'j}^{rr'}D_l^{j'j} \\
& + 2\sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\chi^{r1}\sqrt{(1-\xi^r)A^{rr'}\Psi^{rr'}\sum_{j=1}^{|J|}\sum_{j'\neq j}^{|J|}\sum_{l=1}^{|L|}P_{j'j}^{rr'}D_l^{j'j}} \\
& + 0.5\sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{i\in\left\{D_{in}\right\}^+}^{|I|}A_i^r(|J|-1) + 0.5\sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{l\in\left\{D_l\right\}^+}^{|L|}A_l^r(1-`x_l) + 0.5\sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{r'\in\left\{E^{rr'}\right\}^+}^{|R|}A^{rr'}
\end{aligned}
\tag{36}
$$

Minimizing Eq. 36 under the constraints of Eqs. 30 and 35 with respect to the average flow rates gives a supply chain network that does not yield a product shortage.

Design Example and Discussions

Urgent customer demand can be treated by fully flexible supply chain design; however, such design is highly costly. In this design example, we will show that managing shortage cost with limited-flexible supply chain may be more cost-effective. Figure 5 illustrates an example production inventory and a distribution system that this study is designed to handle. This example includes 18 plants (rounded rectangles), 14 materials (circles), and 3 production processes (squares). The first three plants involve semicontinuous production processes i1–i3 with material storage units j1–j42, which are different from the example in Ref. 21. Each production plant is assumed to only include one production process for the purpose of this case study. Multiple production processes in a plant can introduce additional costs for a given material flow rate. The number of processes can be increased subsequently according to the procedure suggested in our previous research[17] if required. The other plants include only material storage units j43–j252. Plants p4– p9 correspond to the distribution centers, and the other plants correspond to terminals. Raw material purchasing is not considered in this study because the raw materials of a commodity polymer process, ethylene, propylene, hydrogen, etc., are usually supplied by pipelines (from a naphtha cracker) within the same plant. The number of products (14) is similar to the number of products associated with HDPE reactor. We assume that no transportation deliveries are made from plants p4– p18 (distribution centers and terminals) to plants p1–p3 (production plants), and no multiproduct deliveries are made from plants p1–p9 (production plants and distribution centers) to p10–p18 (terminals). The number of possible transportation routes is 24 for a multiproduct delivery and 471 for a single-product delivery. The supply chain network is owned by six subsidiaries r1–r6. Two horizontally curved dotted lines represent the borders between countries, and three countries are involved. Each country has two subsidiaries: the first country has r1/r2, the second has r3/r4, and the third has r5/r6. Each country uses a single currency, and each subsidiary uses the currency associated with its country. Therefore $\chi^{12}=\chi^{34}=\chi^{56}=1$. The currency of the first country ($) is the numeraire. We assume that arbitrage is not present in the currency exchange, which is $\chi^{rr'}=\frac{1}{\chi^{r'r}}$. The currency exchange rates are set to $\chi^{13}=\chi^{35}=3.16$. The corporate income taxes are set to $\xi^1=\xi^2=0.35$, $\xi^3=\xi^4=0.28$ and $\xi^5=\xi^6=0.07$. The interest rates are set to $\eta^1=\eta^2=0.01$, $\eta^3=\eta^4=0.05$, and $\eta^5=\eta^6=0.11$. The production process setup costs are $158, $482, and $668 for i1– i3. The transportation setup costs for a multiproduct delivery are in the range $52– $279. The transportation setup costs for a single-product delivery are in the range $20– $145. The currency transfer setup costs are $0.5– $5. The product change-over cost is $5.92. The operating and opportunity costs of inventory holding are in the range 2.9–13.5 $/L/year. Product sales and transfer prices are 59– 271 $/L. The major constraints are the absence of a material inventory shortage in response to the most urgent customer demand, and material balance. The production processes are assumed to initiate at day 2, and the transportation processes are assumed to initiate at day 8. The customer demand startup times are selected to fall on days 10, 15, 20, 25, 30, 40, 50, and 60. We assume that all SOTFs are zero for convenience. For the given data, we would like to optimize the sizes of the production/transportation processes and the material storage, as well as optimize the material/currency inventories and the average material/currency flow rates through the routes.

The network design model used to determine the average material/currency flow rates can be formulated as an MINLP. This MINLP model has 519 binary variables, 1,648 continuous variables, and 1,565 equations with 792 nonlinear variables and 260 nonlinear constraints. Binary variables represent the existence of production, transportation, and currency transfer processes. The most nonlinear term is a three-variable term $D_i`x_{ij}\left({}^{*}\omega_i\right)|_{\Omega(j)=0}$ in Eq. 35. The best

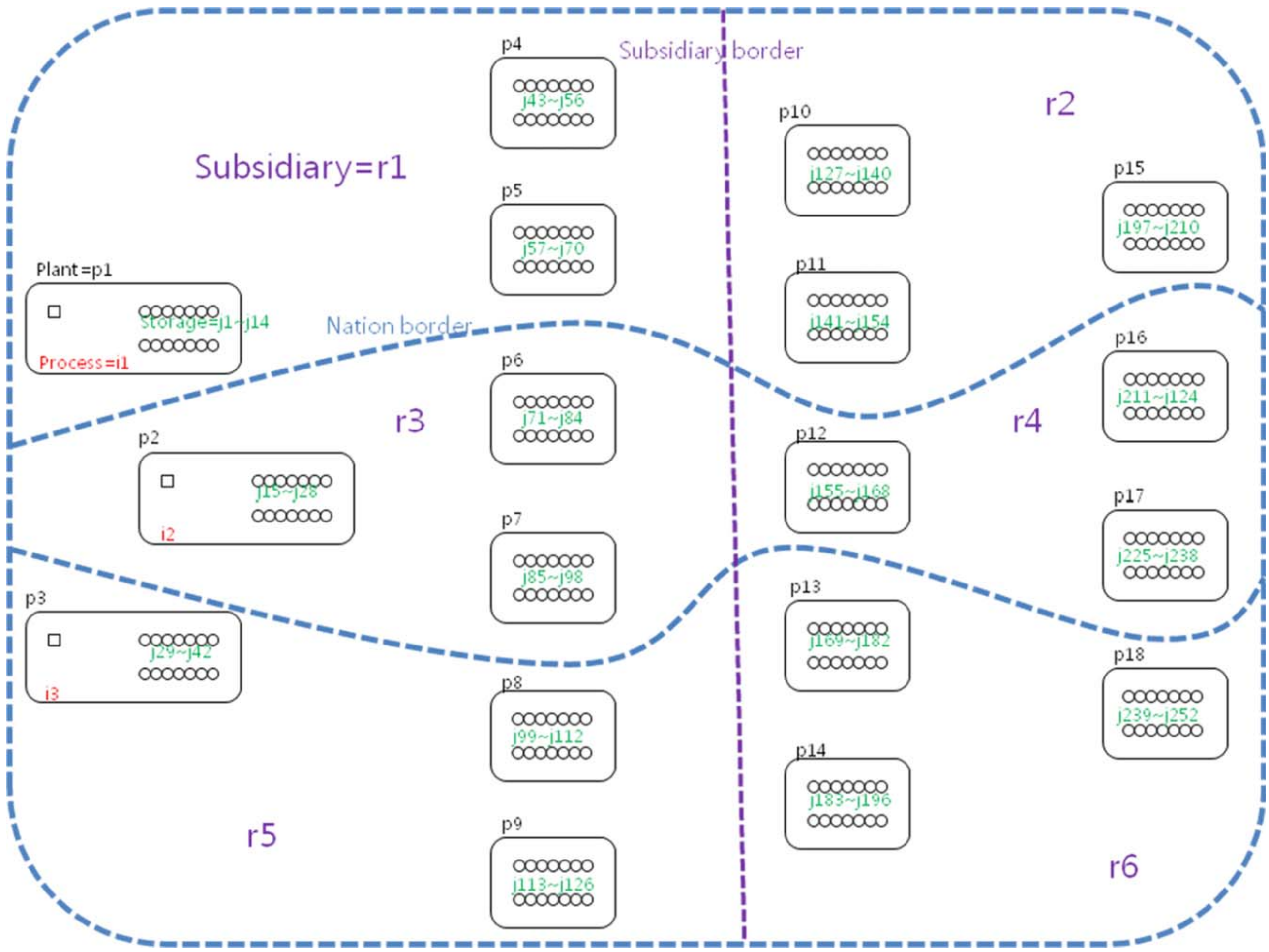

Figure 5. Example supply chain network design.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

commercial global optimizer currently in existence is BARON.[24] Unfortunately; GAMS/BARON cannot solve a problem with this level of MINLP complexity. GAMS/DICOPT 23.8.2 can calculate a local optimum point rapidly only if a suitable initial guess is given. We generated initial guesses for the GAMS/DICOPT using a separable programming technique for a nonseparable function,[21] as summarized in Appendix B posted online at http://myweb.pknu.ac.kr/gbyi/. Separable programming techniques are based on the fact that any nonlinear function can be piecewise approximated by a sufficient number of linear functions. This technique transforms a nonseparable MINLP to a large-scale MILP, which permits us to take advantage of powerful MILP commercial solvers.[25,26] Note that the three-variable terms are segregated into two two-variable terms to reduce the problem size. Each variable may require more than 60 grids to achieve sufficient accuracy[21] in the piecewise linear approximation; however, in this study, we divided separable variables into 10 grids, the variables in $D_l^{ij'}(^*\omega_l)|_{\Omega(j)=0}$ into 50 grids, and all the other variables into 20 grids in order to make function evaluation errors checked at converged point less than 1%. We positioned more grid points where the function becomes stiff in such a way that the interval between grid points increases linearly. The resulting MILP model included 519 binary variables, 555,979 continuous variables (19,839 SOS2 variables), and 28,547 equations. GAMS/XPRESS 22.01 was used as the MILP solver with an Intel(R) Core(TM) i7 CPU X990 @ 3.47 GHz and 24 GB RAM. Note that this CPU has 12 threads, and the parallel (multiple threads) computing capabilities of MILP solver were fully exploited. The total computational times to obtain first feasible solutions were in the range of 3–7 h in the MILP. The first feasible solutions of MILP accepted as initial guesses for succeeding MINLPs were quite far from global optimality, (Current-LP relaxed)/LP relaxed ≊ 49.4–6.9. Succeeding MINLPs with GAMS/DICOPT improved the objective values remarkably within 5 min; (Current-LP relaxed)/LP relaxed ≊ 14.3–2.6. With our limited computational experience, GAMS/XPRESS was more effective than GAMS/CPLEX for parallel computing. Binary variable branching priorities were set higher according to material flow sequence. The rigorous solution of the complete MINLP model introduced here is not practical at this time because of the heavy computational burden; however, given that MILP solution via CPLEX has been speeded up by a factor of over 300 times over the past decade,[27] it can be expected that rigorous solution will be feasible within the next decade. Recent work introduces a branch-and-refine algorithm for the rigorous global optimization of MINLP with square root terms[28] and interval elimination strategy for the rigorous global optimization of MINLP with bilinear terms.[29] Combining these two methods may generate a global optimum for the MINLP introduced in this study; however, it goes beyond the scope of this study.

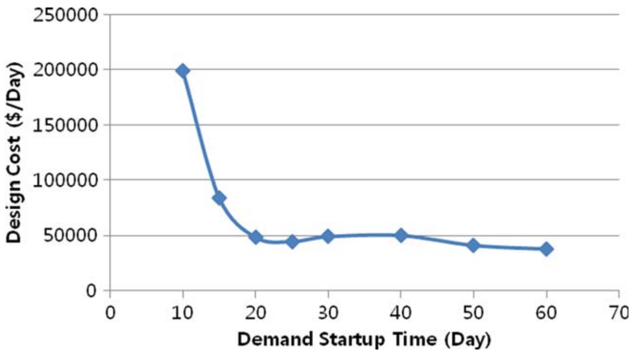

Figure 6. Optimal design cost with respect to demand startup time.

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Figure 6 shows the optimized total network design cost as a function of the customer demand startup time. As the customer demand was delayed, the network design cost decreased until day 20. After day 20, the network design cost remained steady. Therefore, day 20 is a good time for the earliest customer demand to be generated. Customer demand generated earlier than day 20 induces shortage cost. When Solution 4 is selected, the total shortage cost is

$$\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} {}^{*}LCGW(t_m^j, \omega_m^j, v_m^j(0))$$
$$=0.5\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\chi^{r1}\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}H_m^{jr}\right)\left(\sum_{r=1}^{|R|}\chi^{r1}\beta_m^{jr}\right)}{\left(\sum_{r=1}^{|R|}\chi^{r1}\left(\beta_m^{jr}+H_m^{jr}\right)\right)}$$
$$\times D_m^j\left({}^{*}\omega_m^j\right) \tag{37}$$

Equation 37 can be simplified to 0.5 (number of products and customers, inventory holding costs, and shipping batch size) if the backlogging cost is much greater than the oversupplied inventory holding cost. The optimized total network design cost without Eq. 35 included 56% of the design cost predicted with Eq. 35. These results demonstrate that managing the shortage costs in response to urgent customer demand is an essential part of the supply chain network design.

Concluding Remarks

This study seamlessly combines the concepts of backlogging and lost sales costs with the optimal design of a generalized batch-storage network. In previous developments relating to batch-storage network design, the customer demand flow and the product shipping flow were expressed using the same batchwise formulation with known parameter values. In this study, the customer demand flow was assumed to vary continuously, whereas the product shipping flow was batchwise, following the assumption of the classical EOQ model with backlogging or lost sales costs. Note that the cycle time and startup time associated with the product shipping flow were variables that were optimized to minimize the relevant costs. The discrepancies between the two flows caused repeated shortage and oversupply of the product, which incurred costs at different rates. The optimization model that included these additional costs provided optimal startup and cycle times for the product shipping flows. The result obtained in this study based on the generalized batch-storage network formulation converged to the solution to a classical EOQ model as a special case, suggesting the existence of many additional solutions, depending on the network conditions. The product shipping cycle time computed in this study was smaller than that optimized by the classical EOQ model. The initial inventory and startup/cycle times for the purchase, production, and transportation processes jointly influenced the optimal solution to the shortage costs as well as the average flow rates through the network. The shortage costs should be measured on a customer-by-customer basis because the optimal supply chain network structure can depend strongly on the individual customer shortage costs. An example network design showed that imposing the no shortage condition significantly increased the design cost, and, thus, the requirement to actively manage the shortage is unavoidable over a portion of the supply chain network operations.

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Notation

Subscripts

i = index of production process
k = index of raw material vendor
l = index of transportation process
m = index of customer
n = index of task in production process or parcel in transportation process

Superscripts

j = storage index
r = index of currency in subsidiary

b_k^{jr} = annualized capital cost of raw material purchasing facility, currency/L/year, parameter
b_m^{jr} = annualized capital cost of finished product sales facility, currency/L/year, parameter
b_i^r = annualized capital cost of production process i paid by currency r, currency/L/year, parameter
b_l^r = annualized capital cost of transportation process l paid by currency r, currency/L/year, parameter
b^{jr} = annualized capital cost of storage facility j paid by currency r, currency/L/year, parameter
A_k^{jr} = setup cost of feedstock materials, currency/batch, parameter
A_{in}^r = setup cost of task n in noncontinuous process i, currency/batch, parameter
A_{ln}^r = setup cost of parcel n in transportation process l, currency/batch, parameter
$A^{rr'}$ = setup cost of currency transfer, currency/transaction, parameter
B_k^j = raw material order size, L/batch, variable
B_{in} = batch size of task n production process i, L/batch, variable
B_m^j = final product delivery size, L/batch, parameter
B_{ln} = batch size of parcel n transportation process l, L/batch, variable
$C^r(0)$ = initial cash inventory of currency r, L, variable
$C^r(t)$ = cash inventory of currency r at present time t, L, variable
$\overline{C^r}$ = average level of currency inventory, L, variable
$\underline{C^r}$ = lower level of currency inventory, L, variable
$\overline{D_k^j}$ = average material flow of raw material supply, L/year, variable
$\left\{D_k^j\right\}^+$ = the set of index j that has positive value of D_i, variable
$D_{ln}^{jj'}$ = average material flow of customer demand, L/year, parameter
D_{in} = average material flow through task n in production process i, L/year, decision variable

D_i = average material flow through production process i, L/year, decision variable

$\{D_i\}^+$ = the set of index i that has positive value of D_i, variable

$D_{ln}^{jj'}$ = average material flow rate from storage j to storage j' via parcel n in transportation process l, L/year, decision variable

$\{D_{ln}^{jj'}\}^+$ = the set of index l or n that has positive value of $D_{ln}^{jj'}$, variable

E_Σ^r = average currency flow rate of profit after tax, currency/year, variable

E_T^r = average currency flow rate of corporate income tax, currency/year, variable

$E^{rr'}$ = average currency flow rate of currency transfer from r to r', currency/year, variable

$\{E^{rr'}\}^+$ = the set of index r, r' that has positive value of $E^{rr'}$, variable

f_{in}^j = feedstock composition of task n in production unit i, parameter

g_{in}^j = product yield of task n production unit i, parameter

h^{jr} = annual inventory operating cost, currency/L/year, parameter

H_m^{jr} = annual oversupplied inventory holding cost of product j to customer m paid by currency r, currency/L/year, parameter

I = production process set, parameter

$I(r)$ = production process subset owned by the subsidiary that uses currency r, parameter

J = material storage set, parameter

$J(r)$ = material storage subset owned by the subsidiary that uses currency r, parameter

$K(j)$ = raw material supplier set for storage j, parameter

$M(j)$ = consumer set for storage j, parameter

$N(i)$ = task set for production process i, parameter

$N(l)$ = parcel set for transportation process l, parameter

P_k^{jr} = price of raw material j from supplier k paid by currency r, currency/L, parameter

P_m^{jr} = sales price of finished products to customer m paid by currency r, currency/L, parameter

$P_{j'j}^{rr'}$ = transfer price represented by currency r and transferred from currency storage $r \in R(j)$ to currency storage $r' \in R(j')$ for the material transported from storage $j' \in J(r')$ to storage $j \in J(r)$, currency/L, parameter

PSW = the first type of periodic square wave function defined by Eq. 4, variable

PSW' = the second type of periodic square wave function defined by Eq. 5, variable

$\overline{PSW}$ = average level of the first type of periodic square wave function defined in Table 1 of Ref. 10, variable

$\overline{PSW'}$ = average level of the second type of periodic square wave function defined in Table 1 of Ref. 10, variable

$\overline{\overline{PSW}}$ = upper bound of the first type of periodic square wave function defined in Table 1 of Ref. 10, variable

$\overline{\overline{PSW'}}$ = upper bound of the second type of periodic square wave function defined in Table 1 of Ref. 10, variable

$\underline{\underline{PSW}}$ = lower bound of the first type of periodic square wave function defined in Table 1 of Ref. 10, variable

$\underline{\underline{PSW'}}$ = lower bound of the second type of periodic square wave function defined in Table 1 of Ref. 10, variable

R = currency set, parameter

t_m^j = startup time of product shipping, year, variable

$\underline{t_m^j}$ = startup time of customer demand, year, parameter

$\grave{t}_i$ = startup time of feedstock feeding to batch production process i, year, decision variable

$\overline{\overline{\grave{t}_i}}$ = upper bound of startup time of feedstock feeding to batch production process i, year, variable

$\underline{\underline{\grave{t}_i}}$ = lower bound of startup time of feedstock feeding to production process i, year, decision variable

t'_i = startup time of product discharging from production process i, year, variable

t_k^j = startup time of raw material purchasing, year, variable

$\grave{t}_l$ = startup time of loading of transportation process n, year, decision variable

$\overline{\overline{\grave{t}_l}}$ = upper bound of startup time of loading of transportation process n, year, variable

$\underline{\underline{\grave{t}_l}}$ = lower bound of startup time of loading of transportation process n, year, variable

t'_l = startup time of unloading of transportation process n, year, variable

$\grave{t}_{ln}$ = startup time of loading of parcel n in transportation process l, year, variable

t'_{ln} = startup time of unloading of parcel n in transportation process l, year, variable

$t^{rr'}$ = startup time of currency transfer, year, decision variable

$\Delta t_{in}(.)$ = the time lag of task n production process i, year, variable

Δt_k^j = disbursement drifting time of account payables, year, parameter

Δt_m^j = collection drifting time of account receivables, year, parameter

Δt_{ln} = the time lag of parcel n in transportation process l, year, parameter

$v_m^j(0)$ = initial inventory dedicated to customer m, L, variable

$\overline{\overline{V^j}}$ = upper bound of material inventory holdup, L, variable

$\underline{\underline{V^j}}$ = lower bound of material inventory holdup, L, variable

$V^j(t)$ = material inventory hold-up, L, variable,

$V^j(0)$ = initial material inventory holdup, L, parameter

$V^j(t)$ = time averaged material inventory holdup L, variable

x_k^j = storage operation time fraction of purchasing raw materials, parameter

$\grave{x}_{in}$ = storage operation time fraction of feeding to task n in production process i, variable

x'_{in} = storage operation time fraction of discharging from task n in production process i, variable

x_m^j = storage operation time fraction of product shipping, parameter

$\grave{x}_{ln}$ = storage operation time fraction of loading to parcel n in transportation process l, parameter

x'_{ln} = storage operation time fraction of unloading from parcel n in transportation process n, parameter

y_{in} = duration of task n in production process i divided by cycle time

$\grave{y}_{ln}$ = defined by Eq. 2, parameter

y'_{ln} = defined by Eq. 2, parameter

Greek letters

α_m^{jr} = lost sales cost of product j to customer m paid by currency r, currency/L/year, parameter

β_m^{jr} = backlogged inventory holding cost of product j to customer m paid by currency r, currency/L/year, parameter

β^{jr} = initial inventory preparation cost of product j paid by currency r, currency/L/year, parameter

ξ^r = corporate income tax rate paid by currency r, currency/currency, parameter

$\chi^{rr'}$ = foreign currency exchange rate from r to r', currency/currency, parameter

χ^{r1} = foreign currency exchange rate from r to 1, currency/currency, parameter

γ^{jr} = opportunity cost of inventory holding paid cost of material j by currency r, currency/L/year, parameter

η^r = opportunity cost of currency holding paid by currency r, currency/currency/year, parameter

θ^{jr} = $\equiv \frac{(1-\xi^r)h^{jr}+\gamma^{jr}}{2}+(1-\xi^r)b^{jr}$, parameter

$\pi_{ln}^{j'jr}$ = customs duty rate of the material moved from storage j' to storage j by parcel n transportation process l, paid by currency r, currency/L, parameter

π_i^r = operating cost rate of production process i, paid by currency r, currency/L, parameter

ω_m^j = cycle time of product shipping, year, variable

ω_k^j = cycle time of raw material purchasing, year, variable

ω_i = cycle time of production process i, year, variable

ω_l = cycle time of transportation process l year, variable

$\omega^{rr'}$ = cycle time of currency transfer from r to, year, variable

$\Omega(j)$ = a term in Lagrangian multiplier defined in Table 1–4, parameter

Ξ_i = intermediate variable defined by Eq. 32, variable

Ξ_l = intermediate variable defined by Eq. 34, variable

Ψ_{in}^r = aggregated cost for task n in production process i defined by Eq. 18, currency/L/year, parameter

Ψ_k^{jr} = aggregated cost for raw material purchase defined by Eq. 15, currency/L/year, parameter

Ψ_m^j = aggregated cost for raw material purchase defined by Eq. 22, currency/L/year, parameter

$\Psi^{rr'}$ = aggregated cost for currency transfer from currency storage r to currency storage defined by Eq. 24, currency/currency/year, parameter

$\Psi_{ln}^{jj'r}$ = aggregated cost for parcel n in transportation process n to move from storage j to storage j' paid by currency r, defined by Eq. 20, currency/L/year, parameter

Special functions

int[.] = truncation function to make integer
res[.] = positive residual function to be truncated
$|X|$ = number of elements in set X

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